EFFECT OF MULTIPLE STENOSIS ON BLOOD FLOW IN HUMAN ARTERY

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Abstract: This paper considers the effect of multiple stenosis on blood flow in human arteries taking blood as non-Newtonian fluid. Constraints of blood flow in human arteries are known as stenosis which normally ends up into hypertension/stroke and heart failure. Also the deposits of cholesterol on the arterial wall and proliferation of connective tissues are responsible for the abnormal growth in the lumen of an artery. The expressions for the flow characteristics, the pressure gradient, wall shear stress, velocity and volumetric flow rate are obtained in the course of this study. Observations revealed that: as Hematocrit and Viscosity increases, the arterial wall shear stress decreases which indicates increase in human heart pressure. However, increase in Hematocrit (H) and the length of the artery is inversely proportional to the arterial wall shear stress, this signifies the damage of veins around the arteries. The blood pressure gradient increase which is directly proportional to the length of the artery and Hematocrit, this suggests clotting of blood in human heart which can lead to death.

Keywords: Blood flow, blood viscosity, stenosis, wall shear stress

Introduction

Atherosclerosis is the most leading causes of death due to heart diseases and the first symptom of atherosclerotic cardiovascular disease of men and women is heart attack, therefore it is very important to study the blood flow through the stenosed arteries. Stenosis formed on the arterial wall due to the accumulation of cholesterols and fats and the abnormal growth of tissue obstructs the blood flow easily. The build-up that results is called plaque which might be the cause of partially or totally blockage of arterial blood flow. The hematocrit is the volume percentage (Vol.%) of red blood cells in the blood. It is normally 45% for men and 40% for women. It is considered an integral part of a persons complete blood count results, along with haemoglobin concentration, white blood cell count and platelet count. The hematocrit blood test determines the percentage of red blood cells (RBC`s) in the blood.

The study of blood flow through human arteries under multistenotic conditions is believed to be one of the most physiological problems because of the fact that the cause and development of many vascular diseases leading to the malfunction of the cardiovascular system have a close relationship with the nature of blood flow and the deformability of the vascular walls. The narrowing of the arterial walls is caused by atheroma, a deposition of fats and fibrous tissue in the arterial lumen. This constriction of the arterial lumen grows inward and restricts the normal flow of blood when the transportation of blood to the region beyond the narrowing is reduced considerably.

Several theoretical and experimental studies, such as the effects of stenosis on blood flow characteristics with respect to time-dependent stenosis on flow through a tube, the flow in locally constricted tubes at low Raynolds numbers, on the coupling and detection of motion between an artery with a localized lesion and its surrounding tissue, and on the steady flow through modelled vascular stenoses (Lee and Fung, 1970; Srivatsava and Rastogi, 2009; Young, 1968) were reported.

Chakravarty and Mandal (1994) studied analytically the unsteady flow behavior of blood in an artery under stenotic condition, by considering the blood to be a non-Newtonian fluid, and taking into account also the viscoelasticity of the blood. Gijsen *et al.* (1999) studied the impact of non-Newtonian properties of blood on the velocity distribution. They made a comparison between the non-Newtonian fluid model and a Newtonian fluid at different Renolds numbers. The comparison states that the character of flow of the non-Newtonian fluid is simulated quite well by using the appropriate Reynolds number.

The work of Onitilo and Usman (2018) investigated the mathematical analysis of blood flow through a stenosed human artery. In their work, it was revealed that increase in Hematocrit and viscosity decreases the arterial wall shear stress, which indicate an increase in human heart pressure. However, increase in Hematocrit (H) and the length of the artery (Z) reduces the arterial wall shear stress, this signifies the damage of veins around the arteries. The blood pressure gradient increases as the length of the artery (Z) and Hematocrit increases; this suggests clotting of blood in human heart which can lead to death.

Mabotuwana *et al.* (2007) observed that there were some early clinical indicators of cardiac ischemia, most notably a change in a person's electrocardiogram. Less well understood, but potentially just as dangerous, was ischemia that develops in the gastrointestinal system. Such ischemia was difficult to diagnose without angiography (an invasive and time-consuming procedure) mainly due to the highly unspecific nature of the disease. Understanding how perfusion is affected during ischemic conditions can be a useful clinical tool which can help clinicians during the diagnosis process.

In the work of Harjeet *et al.* (2010) the increased impedance and shear stress during a narrow catheterized has been analyzed assuming that the flowing blood is represented by a Newtonian fluid. The impedance increases with increasing catheter size and depends on stenosis height. Saktipada and Ratan (2012) worked on a mathematical model which was developed for studying blood flow through a narrow artery with multiple stenosis. Under the consideration of non-Newtonian character of blood a constitutive equation of blood is taken, described by Herschel-Bulkley equation. The problem is investigated by a combined use of analytical and numerical techniques.

In this current research, the effect of multiple stenosis at the wall of the arteries which hinder the five flow of the blood flow was considered.

Mathematical Formulation of Blood Flow

In this paper, the following assumptions are taken into consideration; the flow is an incompressible, non-Newtonian with constant density, and variable viscosity. The schematic diagram of blood flow in the artery is shown in Fig. (1). The problem studied cylindrical coordinate system (r, θ, z) where z axis is taken along the axis of the artery while r and θ are along the radial and the circumferential directions, respectively.

The geometry of the stenosis in one-dimensional form which develops symmetrical about the artery axis but non-symmetric with respect to radial coordinate is given as;

$$\frac{R(z)}{R_0} = 1 - a[(S_L)^{k-1}\{z-(d_1+d_2+d_3+d_4+d_5)\} - \{z-(d_1+d_2+d_3+d_4+d_5)\}^k]$$

$$(d_1+d_2+d_3+d_4+d_5) \le z \le (d_1+d_2+d_3+d_4+d_5)+S_L \quad (1)$$

Where: $a = \frac{\delta}{R_0(S_L)^k}$

Where: $d_1, d_2, \ldots, d_5$ are distance of the stenosis region, z is the length of the artery, δ is the maximum height of the stenosis, R_0 is the radius of the vessel without stenosis and $R(z)$ is radius of the vessel with stenosis.

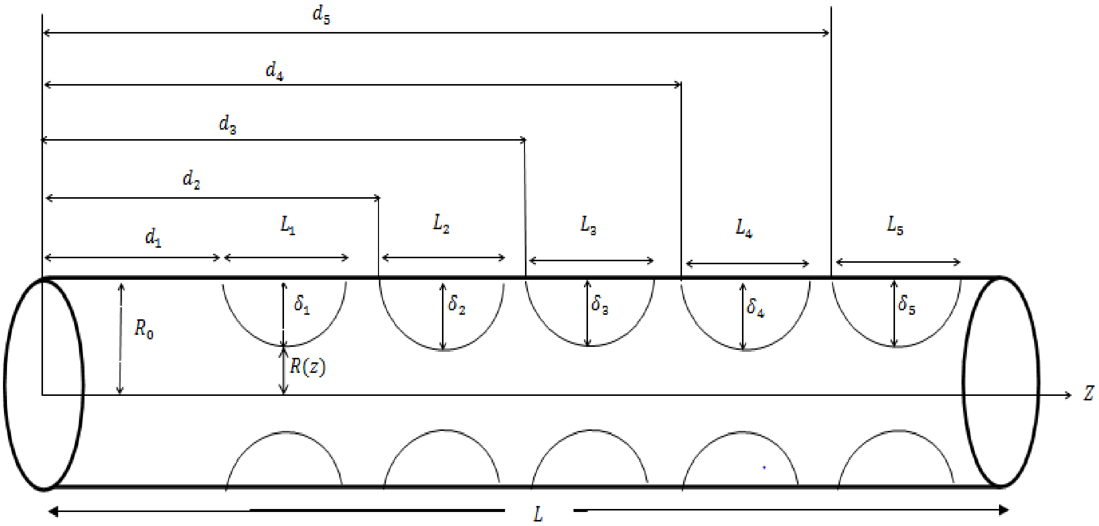

Fig. 1: Schematic diagram of blood vessels with stenosis ($n = 5$)

Consider the one-dimensional equation for the steady and axially symmetric flow of blood through an artery provided with a mild stenosis under the above mentioned assumption is:

$$\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}[r,\tau] = 0\,, \qquad (2)$$

Where: p is the fluid pressure, τ is the shear stress, $(r\tau)$ are the cylindrical polar co-ordinates with z measured along the stenosis axis and r measured normal to the axis of the stenosis. According to Lih (1969), the Einstein coefficient of viscosity of blood was given as:

$$\mu\ (r) = \mu_0[1 + ch(r)], \qquad (3)$$

where μ_0 is the coefficient of viscosity of plasma, $c = 2.5$ and $h^*(r)$ is hematocrit described by the formula

$$h(r) = H\left[1 - \left(\frac{r}{R_0}\right)^n\right] \qquad (4)$$

The boundary conditions are:

$$u = 0, \quad at r = R(z)$$
$$\text{and} \quad \frac{du}{dr} = 0, \quad at r = 0 \qquad (5)$$

Let

$$x = \frac{r}{R_0} \qquad (6)$$

be the radial coordinate, where R_0 is the inner radius of the vessel and x is the arterial wall viscosity.

then on substituting equations (3), into (4); it gives

$$\mu(r) = \left[1 + cH - cH\left(\frac{r}{R_0}\right)^n\right] \qquad (7)$$

on further simplification, equation(7) becomes

$$\mu(r) = \mu_0[b_1 - b_2x^n] \qquad (8)$$

Where $b_1 = 1 + b_2, \qquad b_2 = cH$

The wall shear equation is given as

$$\tau = -\mu(r)\frac{du}{dr} \qquad (9)$$

and on substituting equation (8) into (9), we have

$$\tau = -\mu_0(b_1 - b_2x^n)\frac{du}{dr} \qquad (10)$$

And on further utilization of equation (6) in equation (10), we get

$$\tau = -\mu_0(b_1 - b_2x^n)\frac{du}{dx}\cdot\frac{1}{R_0} \qquad (11)$$

Substituting equation(8) and (11) into equation (2) yields

$$\frac{\partial p}{\partial z} = \frac{\mu_0}{R_0^2}\frac{1}{x}\frac{d}{dx}[x(b_1 - b_2x^n)]\frac{du}{dx} \qquad (12)$$

Multiplying equation (12) by $\frac{R_0^2}{\mu_0}$, gives:

$$\frac{R_0^2}{\mu_0}\frac{\partial p}{\partial z} = \frac{1}{x}\frac{d}{dx}\left[x(b_1 - b_2x^n)\frac{du}{dx}\right] \qquad (13)$$

The boundary condition in equation (5) becomes

$$u = 0, \quad at x = \frac{R(z)}{R_0}$$
$$and \quad \frac{du}{dx} = 0, \quad at x = 0 \qquad (14)$$

Equation (13) and the boundary conditions in equations(14) are applicable only when $n \ge 2$.

Method of Solution

Equation (13) is solved with the help of boundary conditions (14), which gives:

$$\frac{d}{dx}\left[x(b_1 - b_2x^n)\frac{du}{dx}\right] = \frac{xR_0^2}{\mu}\frac{dp}{dz} \qquad (15)$$

Integration equation (15) gives

$$x(b_1 - b_2x^n)\frac{du}{dx} = \frac{x^2R_0^2}{2\mu}\frac{dp}{dz} + C \qquad (16)$$

Where C is the constant of integration

Noting that $\frac{du}{dx} = 0,\ at x = 0,$ then $c = 0. \qquad (17)$

Therefore, equation (16) becomes

$$x(b_1 - b_2x^n)\frac{du}{dx} = \frac{x^2R_0^2}{2\mu}\frac{dp}{dz} \qquad (18)$$

Solving equation (18) with the boundary condition in equation (14) analytically we have

$$u(x) = \frac{b_1}{2b_2}\left[\left(\frac{R(z)}{R_0}\right)^{-2} - \frac{1}{x^2}\right] + \frac{R_0^2}{2\mu_0 b_2}\frac{dp}{dz}\left[\frac{1}{x} - \left(\frac{R(z)}{R_0}\right)^{-1}\right] \qquad (19)$$

which is the velocity of the blood.

The volumetric flow rate Q is given by:

$$Q = 2\pi R_0 \int_0^{R/R_0} x\,u(x)dx \qquad (20)$$

Hence, substituting equation (19) into equation (20) yields

$$Q = \frac{\pi R_0 b_1}{2b_2} - \frac{\pi R_0 b_1}{b_2} log\left(\frac{R(z)}{R_0}\right) + \frac{\pi R_0^2}{\mu_0 b_2} R(z)\frac{dp}{dz} \qquad (21)$$

If Q_0 is the flow rate of plasma fluid in the unconstricted tube, then

$$Q_0 = -\frac{\pi R_0^4}{8\mu}\left(\frac{dp}{dz}\right)_0 \qquad (22)$$

Let $\frac{Q}{Q_0} = 1;$ then $Q = Q_0$

that is

$$\frac{\pi R_0^2}{\mu_0 b_2} R(z)\frac{dp}{dz} = \frac{\pi R_0 b_1}{b_2} log\left(\frac{R(z)}{R_0}\right) - \frac{\pi R_0 b_1}{2b_2} - \frac{\pi R_0^4}{8\mu}\left(\frac{dp}{dz}\right)_0 \qquad (23)$$

Putting $S_L = 1,\ k = 2,\ d_1 = {}^1/_2,\ d_2 = {}^1/_4,\ d_3 = {}^1/_8,\ d_4 = {}^1/_{16}, d_5 = {}^1/_{32},\ b = {}^1/_2$ into equation (1), it becomes

$$\frac{R(z)}{R_0} = 1 - \frac{1}{2}\left[1\{z - ({}^1/_2 + {}^1/_4 + {}^1/_8 + {}^1/_{16} + {}^1/_{32})\} - \{z - ({}^1/_2 + {}^1/_4 + {}^1/_8 + {}^1/_{16} + {}^1/_{32})\}^2\right] \qquad (24)$$

Further simplification of equation (24) yields

$$\frac{R(z)}{R_0} = \left[\frac{1024z^2 - 3008z + 4001}{2048}\right] = \frac{\varphi}{2048} \qquad (25)$$

Where $\varphi = 1024z^2 - 3008z + 4001$ (26)

Taking logarithm of both sides of equation (25) gives

$$log\left(\frac{R(z)}{R_0}\right) = log\left(\frac{\varphi}{2048}\right) \qquad (27)$$

Substituting equations(25) and (27) into equation (23), yields

$$\frac{{}^{dp}/_{dz}}{\left({}^{dp}/_{dz}\right)_0} = \frac{2048\mu_0 b_1}{\varphi R_0^2} log\left(\frac{\varphi}{2048}\right)\frac{1}{\left({}^{dp}/_{dz}\right)_0} - \frac{1024\mu_0 b_1}{\varphi R_0^2}\cdot\frac{1}{\left({}^{dp}/_{dz}\right)_0} - \frac{256 b_2 \mu_0 R_0}{\varphi\mu} \quad (28)$$

Substituting $\frac{1}{\left({}^{dp}/_{dz}\right)_0} = \frac{\pi R_0^4}{8\mu Q_0}$ into equation (28) gives

$$\frac{{}^{dp}/_{dz}}{\left(\frac{dp}{dz}\right)_0} = \frac{256 b_2 R_0}{\varphi\mu}\left[-\frac{\pi b_1 \mu_0 R_0}{\mu b_2} log\left(\frac{\varphi}{2048}\right) + \frac{\pi b_1 \mu_0 R_0}{2\mu b_2} - \frac{\mu_0}{\mu} Q_0\right] \qquad (29)$$

Putting $Q = Q_0 and \mu = \mu_0$ into equation (29) yields

$$\frac{{}^{dp}/_{dz}}{\left(\frac{dp}{dz}\right)_0} = \frac{256 b_2 R_0}{\varphi Q_0}\left[\frac{\pi R_0 b_1}{2b_2} - Q - \frac{\pi R_0 b_1}{b_2} log\left(\frac{\varphi}{2048}\right)\right] \qquad (30)$$

which is the pressure gradient of the blood flow.

The wall shear stress of artery is defined by

$$\tau_w = \mu\left(\frac{du}{dr}\right)_{r=R(z)} \qquad (31)$$

Since $u = \frac{b_1}{2b_2}\left[\left(\frac{R(z)}{R_0}\right)^{-2} - \frac{1}{x^2}\right] + \frac{R_0^2}{2\mu_0 b_2}\frac{dp}{dz}\left[\frac{1}{x} - \left(\frac{R(z)}{R_0}\right)^{-1}\right]$

(32)

From equation (6), $\frac{x}{r} = \frac{1}{R_0}$ (33)

Substituting equation (33) into equation (32) to obtain

$$u = \frac{b_1}{2b_2}\left[\frac{r^2}{x^2(R(z))^2} - \frac{1}{x^2}\right] + \frac{R_0^2}{2\mu_0 b_2}\frac{dp}{dz}\left[\frac{1}{x} - \frac{r}{xR(z)}\right] \qquad (34)$$

Differentiating equation (34) yields

$$\frac{du}{dr} = \frac{b_1 r}{b_2 x^2 (R(z))^2} + \frac{R_0^2}{2\mu_0 b_2}\frac{dp}{dz}\left(\frac{-1}{xR(z)}\right) \quad (35)$$

Substituting equation (35) into equation (31) gives

$$\tau_w = \mu\left[\frac{b_1 r}{b_2 x^2 (R(z))^2} - \frac{R_0^2}{2\mu_0 b_2}\frac{dp}{dz}\left(\frac{1}{xR(z)}\right)\right] \qquad (36)$$

Putting $r = R(z)$, into equation (36) to obtain

$$\tau_w = \mu\left[\frac{b_1}{b_2 x^2 r} - \frac{R_0^2}{2\mu_0 b_2}\frac{dp}{dz}\left(\frac{1}{xr}\right)\right] \qquad (37)$$

From equation (8) $\mu = \mu_0(b_1 - b_2 x^n)$

Putting $n = 5$ into equation (8) gives

$$\mu = \mu_0(b_1 - b_2 x^5) \qquad (38)$$

Substituting equation (38) into equation (37) to obtain

$$\tau_w = \frac{\mu_0}{x^3}(b_1 - b_2 x^5)\left[\frac{b_1 x}{b_2 r} - \frac{R_0^2}{2\mu_0 b_2 r}\cdot\frac{dp}{dz}\right] \qquad (39)$$

Since $\frac{1}{R_0} = \frac{x}{r}$, therefore equation (39) becomes

$$\tau_w = \frac{\mu_0}{x^3}(b_1 - b_2 x^4)\left[\frac{b_1}{b_2 R_0} - \frac{xR_0}{2\mu_0 b_2}\frac{dp}{dz}\right] \qquad (40)$$

If τ_N is the shear stress of plasma fluid at the normal artery wall, then it is defined as

$$\tau_N = \left(-\frac{R_0}{2}\right)\left({}^{dp}/_{dz}\right)_0 \qquad (41)$$

Therefore the shear stress is

$$\tau = \frac{\tau_w}{\tau_N} \qquad (42)$$

$$\tau = \frac{2\mu_0}{R_0}\left\{\left(b_2 x^2 - \frac{b_1}{x^3}\right)\left[\frac{b_1}{b_2 R_0\left({}^{dp}/_{dz}\right)_0} - \frac{xR_0}{2\mu_0 b_2}\frac{{}^{dp}/_{dz}}{\left({}^{dp}/_{dz}\right)_0}\right]\right\} \qquad (43)$$

But $r = R(z)$, and $x = \frac{r}{R_0} = \frac{R(z)}{R_0} = \frac{\varphi}{2048}$ (44)

$$\tau = \frac{2\mu_0}{R_0}\left(\left(\frac{\varphi}{2048}\right)^2 b_2 - \left(\frac{2048}{\varphi}\right)^3 b_1\right)\left[\frac{b_1}{b_2 R_0\left({}^{dp}/_{dz}\right)_0} - \frac{\varphi R_0}{40692\mu_0 b_2}\cdot\frac{{}^{dp}/_{dz}}{\left({}^{dp}/_{dz}\right)_0}\right] \qquad (45)$$

Results and Discussion

The variation of wall shear stress and pressure gradient along the length of artery and viscosity for different values of Hematocrit (H) of red blood cell when $n = 5$ (when the number of stenosis is five).

The variation of wall shear stress and pressure gradient along the length of human artery and blood viscosity (n=5) for different values of Hematocrit (H) of red blood cell Fig. 2 shows variation of wall shear stress and blood viscosity (n=5) for different hematocrit (H) . Again the wall shear stress reduces when the Hematocrit (H) of red blood cell level increases with increase of blood viscosity (n=5).

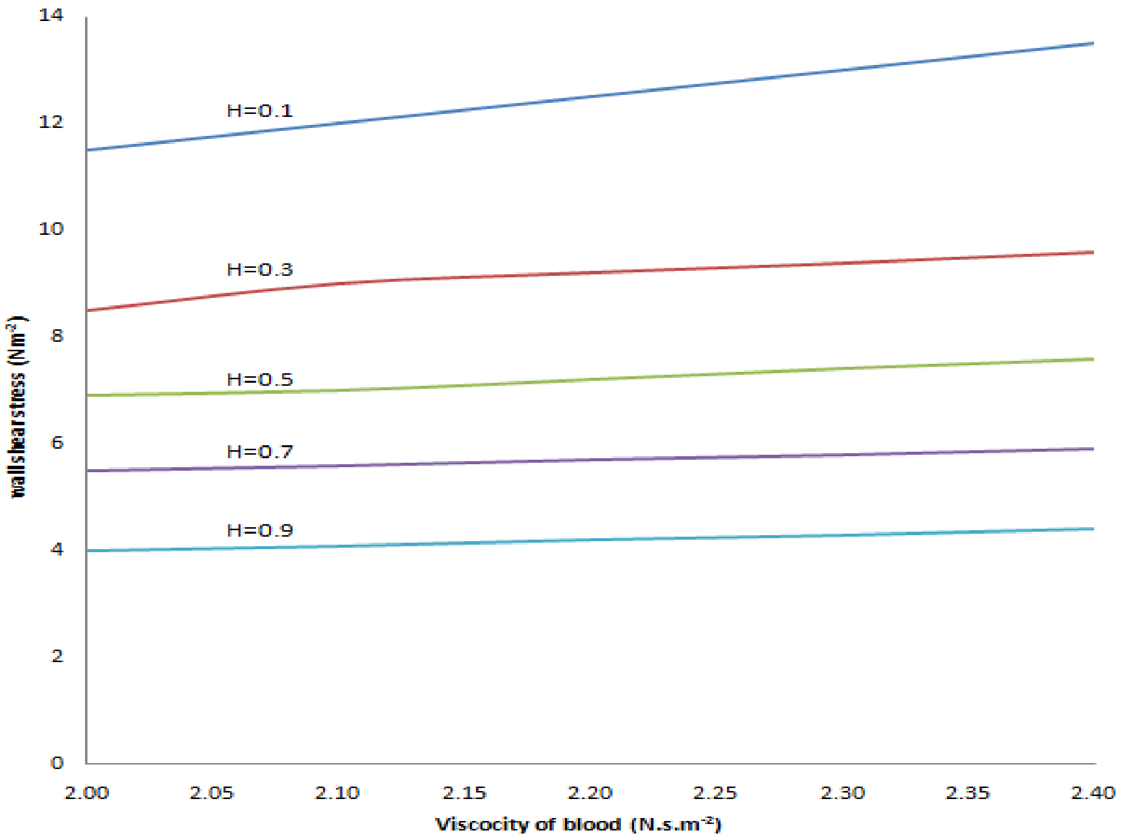

Fig. 2: Variation of wall shear stress along blood viscosity (μ) for different values of hematocrit (H) with the length of the artery Z=1, $n = 5$

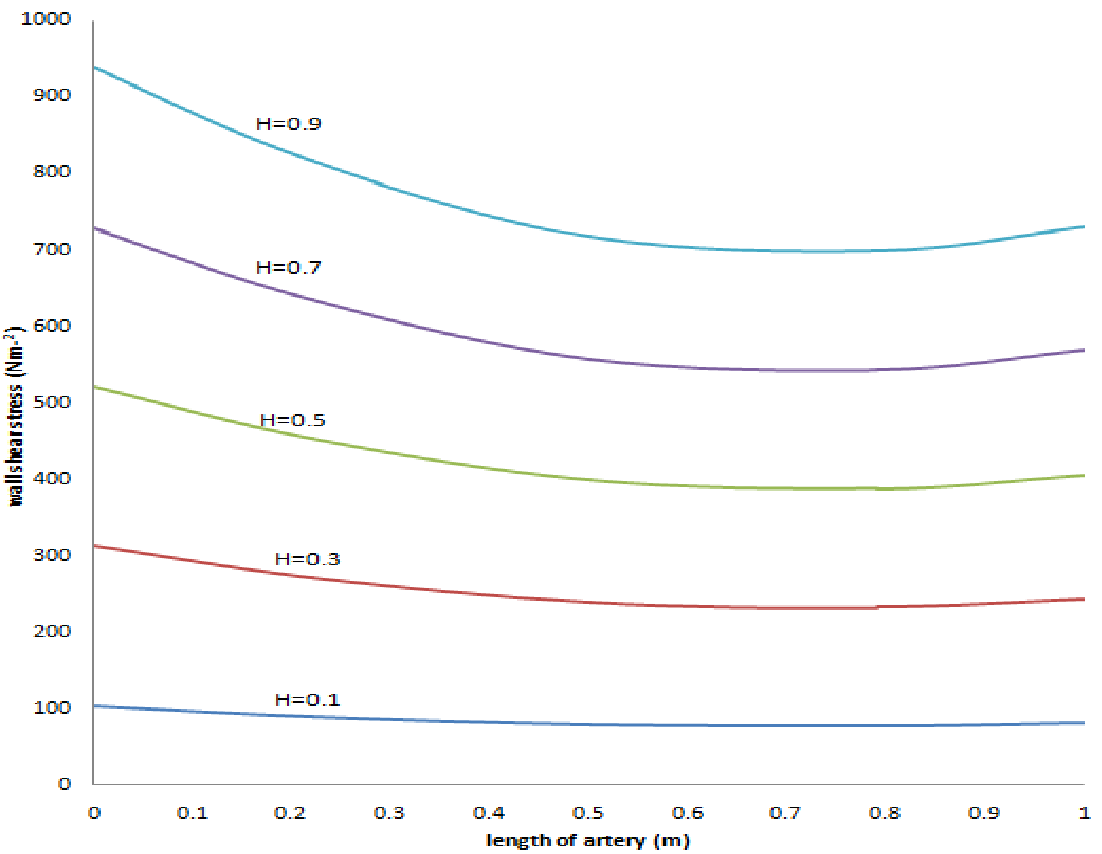

Fig. 3: Variation of wall shear stress along the length of artery for different values of (H) with viscosity (μ) = 2, when $n = 5$

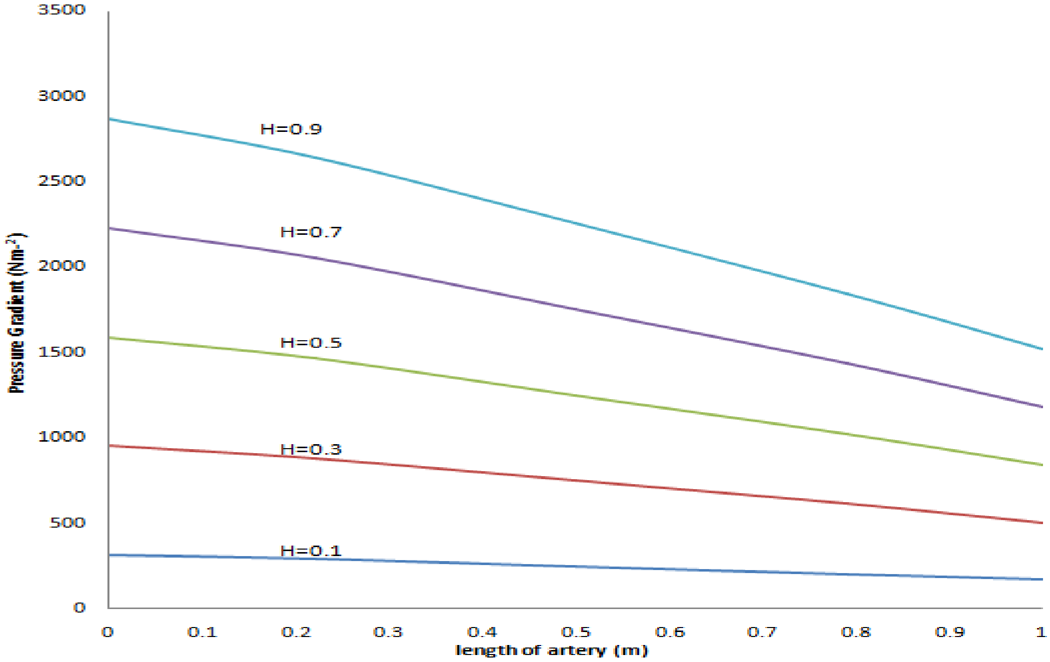

Fig. 4: Variation of pressure gradient along the length of artery for different values of Hematocrit (H) of red blood cell, when $n = 5$

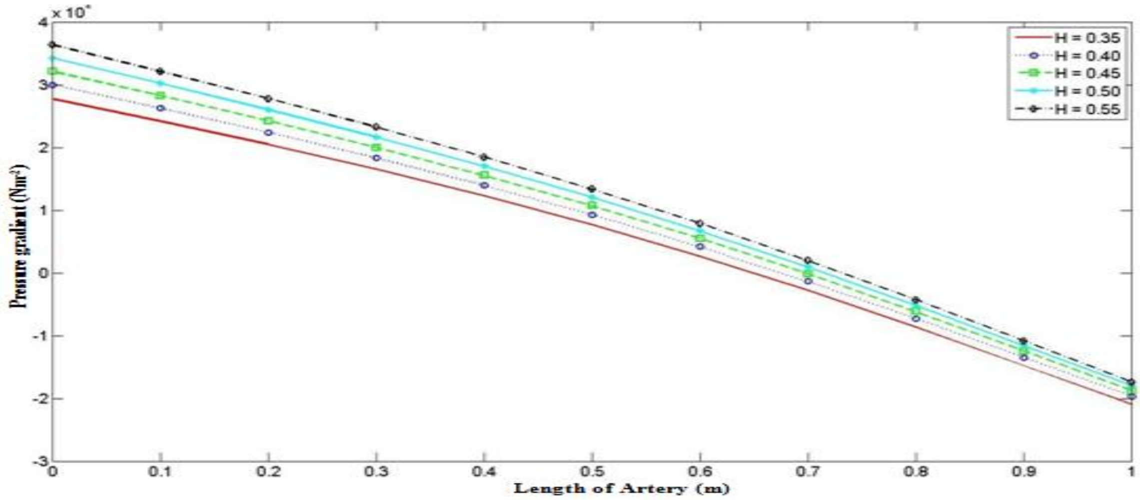

Fig. 5: Variation of wall shear stress along length of artery (Z), when $n = 5$

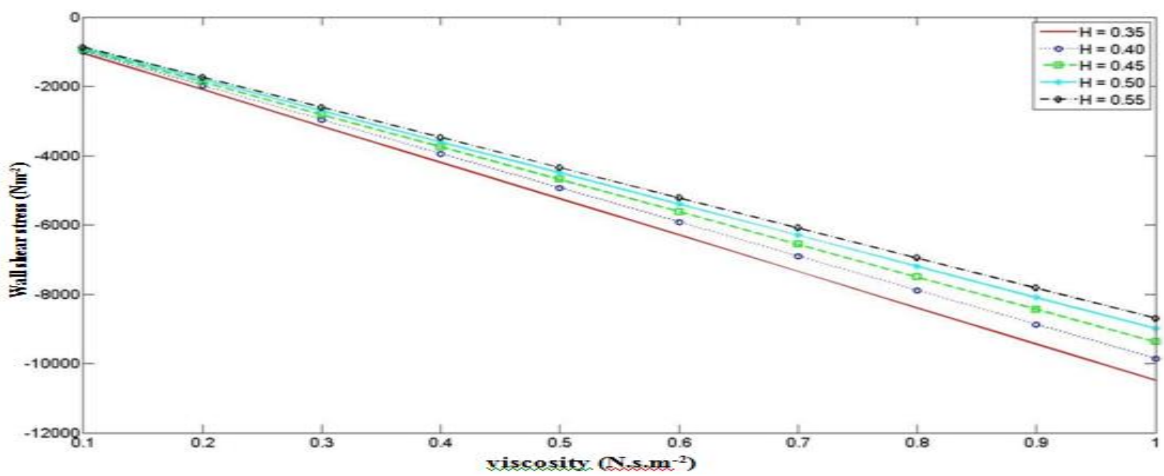

Fig. 6: Variation of pressure gradient along viscosity (μ), when $n = 5$

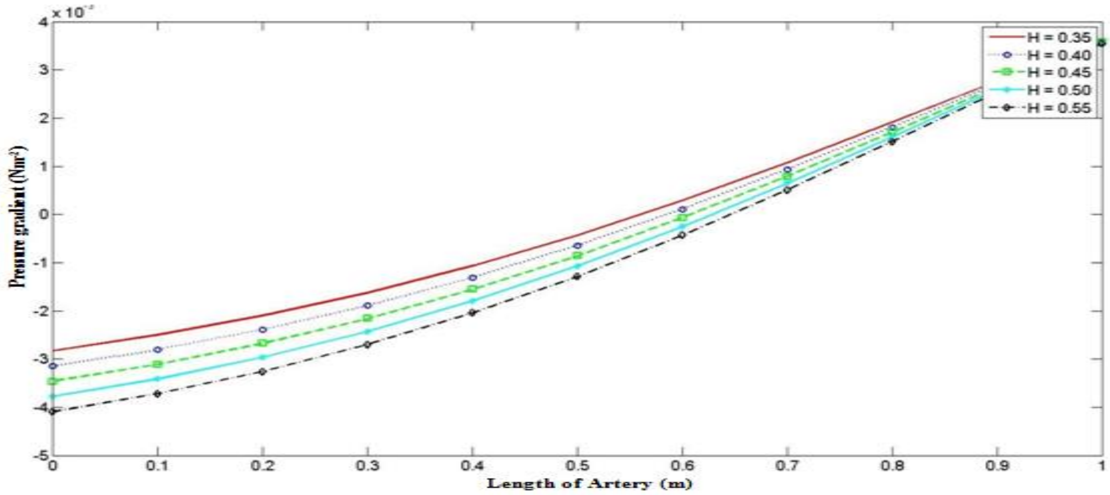

Fig. 7: Variation of pressure gradient along length of artery (Z), when $n = 5$

In the same dimension Figs. 3 and 5 reiterated that the wall shear stress of human artery decreases with increase in viscosity of the blood (n=5) for increase in hematocrit (H) because of the obstruction in the human artery due to stenosis presence in the human artery. Figs. 4, 6 and 7 shows the variation of pressure gradient along the length of human artery with blood viscosity (n=5) for different Hematocrit (H) of red blood cell. At this stage, the pressure gradient increases rapidly for increase in the blood viscosity (n=5) when the hematocrit (H) increases . This is dangerous because the systolic pressure will rise and diastolic pressure will become too low and this result in high blood pressure or hypertension that can kill the cells in human brain that bring about stoke. Also it can make the muscles of the human heart to die and heart failure or heart attack will be the end result when the high blood pressure or hypertension is not treated.

In conclusion, the results obtained indicates that increase in the height and number of stenosis in the artery could result in high blood pressure which can get to a critical region that is beyond control, which can cause bleeding or clotting of the blood which is very dangerous to human heart/brain as it can result into heart failure or stroke.

Conflict of Interest

The author declares that there is no conflict of interest related to this work.

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APPENDIX

$R(z)$	Radius of the artery at stenosed portion
R_0	Radius of the artery
L	Length of the artery
δ	The maximum height of the stenosis
z	Axial coordinate
p	The fluid pressure

(r, z)	The cylindrical polar co-ordinate with z measured along the axis of the stenosis
H	Hematocrit (H) of red blood cell
μ	Viscosity of the fluid
u	Velocity of the fluid
Q	Volumetric flow rate
τ	Wall shear stress
x	Arterial wall viscosity